

Patterns

(a modified IMP Unit)

Number and Operations Unit for High School

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These 15 days address four algebra standards for High School mathematics, with a focus on number and operations. This table provides the links between which standards are addressed in the given activities.

Page	Days	Lesson	MN Standard	Sample MCA Question
3 - 4		Pre-Test		
5	1	The Broken Eggs	<p>9.2.3.4 Add, subtract, multiply, divide and simplify algebraic fractions.</p> <p>9.2.4.8 Assess the reasonableness of a solution in its given context and compare the solution to appropriate graphical or numerical estimates; interpret a solution in the original context.</p>	See 11th Grade Item Sampler #3, 7, 11 & 12
6	2 & 3	Marcella's Bagels		
7 - 8	4	1-2-3-4 Puzzle		
9	5	Uncertain Answers		
10 - 11	6 & 7	Chefs' Hot & Cold Cubes		
12	8 & 9	Checkerboard Squares	<p>9.2.2.4 Express the terms in a geometric sequence recursively & by giving an explicit (closed form) formula, and express the partial sums of a geometric series recursively.</p>	See 11th Grade Item Sampler #7, 11
13	10	Consecutive Sums		
14	11	Add It Up		
15 - 16	12	That's Odd	<p>9.2.2.1 Represent and solve problems in various contexts using linear and quadratic functions.</p>	See 11th Grade Item Sampler #9, 14, 18, 20
17	13	A Fractional Life	<p>9.2.3.4 Add, subtract, multiply, divide and simplify algebraic fractions.</p>	See 11th Grade Item Sampler #3 & 8
18	14	Chef Divisions		
19	15	From One to N	<p>9.2.2.4 Express the terms in a geometric sequence recursively & by giving an explicit (closed form) formula, and express the partial sums of a geometric series recursively.</p>	See 11th Grade Item Sampler #3, 7, 11 & 12
20 - 22		Post Test		

Short Answer.

1. Explain each of the following problems in terms of the model of hot and cold cubes. Each explanation should include a statement of how the temperature changes overall.

a) $-4 - +9$

b) $-4 \cdot +5$

2. Does $-3 - +8 = -3 + -8$? Explain why or why not using the hot and cold cubes.
3. The supervisor of a community garden project organizes volunteers to help dig out weeds. The more people they have, the more weeds get pulled. The results are better than one might think, although one person will pull only two bags a day, two people will pull five bags a day, and three people will pull eight bags a day. The garden must be cleared of winter weeds. The supervisor estimates that there are 30 bags worth of weeds to be pulled. How many volunteers are needed to get the job done in a day?

a) Make an in/out table.

b) Solve the problem.

4. Simplify each expression.

a) $6! - (9 - 5)^2$

b) $18 - 4(3) + 2^5$

c) $\left(100 - (3 \cdot (11 - 7) + \frac{2 + (5(-10))}{3(6-2)})\right)$

5. The product of two even numbers will always be even.

If you think this is true, explain why. If you think it is false, provide a counterexample.

Multiple Choice.

6. What set of numbers is represented by the summation?

$$\sum_{i=5}^9 (n + 2)$$

- a) $5 + 6 + 7 + 8 + 9$ b) $7 + 8 + 9 + 10 + 11$ c) $1 + 2 + 3 + 4 + 5$ d) $10 + 12 + 14 + 16 + 18$

7. Let n be a number such that when divided by 2, 3, and 4 it has a remainder of 1 but when divided by 5 there is no remainder. What number is n ?

- a) 11 b) 12 c) 20 d) 25

8. What rule could describe the following In-and-Out Table?

In	Out
0	20
3	17
8	12
15	5

- a) $y = -3x$ b) $y = -x + 20$ c) $y = -3x - 20$ d) $y = -5x + 17$

Lesson 1 - The Broken Eggs (p. 6)

Objective:

Students will use their knowledge about factors to solve a real-world problem.

Launch: Introduce students to “The Situation.”

“A farmer is taking her eggs to market in her cart. Along the way, she hits a pothole, which jars her cart and spills the eggs.

Though the farmer is unhurt, every egg is broken. She goes to her insurance agent, who asks her how many eggs she had. She doesn’t know, but she does remember some things from various ways she tried packing the eggs.

She knows that when she put the eggs in groups of two, there was one egg left over. When she put them in groups of three, there was also one egg left over. The same thing happened when she put them in groups of four, five, or six. But when she put them in groups of seven, she ended up with complete groups of seven, with no eggs left over.”

“Your task is to answer the insurance agent’s question: What can you figure out from this information about how many eggs the farmer had? Is there more than one possible answer?”

Explore:

Students will work on the problem in groups. If students are stuck, ask them what they have tried, check for understanding of the problem, and/or suggest they try a smaller problem. “Suppose the farmer remembered that only when she put the eggs in groups of either two or five, there was one egg left over. What would be some possibilities for the number of eggs in that situation?”

If any groups find the answer 301, you can urge them to look for other solutions. If groups need further challenges, they can look for a general solution or a description of how to find other solutions, and then for an explanation of how they know their general solution includes all possibilities.

Share:

Students will share ideas in groups to create the best problem statement possible. Ask one or two groups to share their problem statements with the class.

In the discussion, bring out that the problem statement should not simply repeat the problem as originally stated, but should try to focus on the mathematical elements more than the “story” elements. Students should also share their process and solution. The focus should be on discussion of the process, not on having “the” answer.

Summarize:

The teacher should pose the question, “How do you know that 301 is a solution? Do you suspect there are other solutions? Why?” The teacher should summarize the results of the classroom discussion:

- There are multiple routes to a solution. Having a different method for finding or explaining an answer does not mean that it is a new or different answer.
- There may be more than one solution. Always ask yourself, “Is this a problem that has more than one answer?”

Lesson 2 - Marcella's Bagels (p. 16 - 17 and p. 74 - 77)

Objective:

Students will use a variety of methods, such as adding, subtracting, multiplying, dividing and simplifying to solve a real-world problem.

Launch:

Ask student-volunteers to reenact the story of Marcella's Bagels, while the teacher narrates. Direct students to work in their groups on the activity, encourage them to use the materials available to help them think through the problem. If students express that they are beyond using objects like beans or counters, assure them that doing mathematics involves using whatever it takes—pencil and paper, calculators and computers, models and manipulatives — to understand a situation or an idea.

Explore:

Students will work in groups to solve the problem. The problem also lends itself to the powerful strategy of working backward. Students can begin with the number of bagels Marcella has at the end and by undoing each action she took and arriving at the number of bagels she had at the start. At each step, Marcella gives away half of her bagels plus 2, so in reverse she would add 2 and then double the total.

Share:

Discuss the various methods students used to solve the problem.

Ask students:

- **How did you find the answer?**
- **How do you know your answer is correct?**

The two most likely approaches will be (1) guessing the starting amount and then running the problem forward to see if the guess leads to the correct ending amount and (2) working backward.

Summarize:

Students will then read a write-up of this problem and try to focus on how the write was able to communicate how they solved the problem. In their groups, have them answer the following questions and then share with the class.

- How did you find the answer?
- How do you know your answer is correct?
- What is missing?
- What isn't needed?
- How does the process used here differ from the solution?

Lesson 3 - 1-2-3-4 Puzzle (pg. 18)

Objective: Students will gain insight into the need for rules for order of operations and provide additional experience with the algebraic logic of graphing calculators.

Launch: Ask someone to volunteer a numeric expression using each of the numbers 1, 2, 3, and 4 and any operations they would like. Record their suggestion, and ask the class to calculate the result.

The teacher may need to guide a discussion about order of operations and the use of parentheses.

Ask for two or three more expressions, again instructing the class to calculate the results, and then wonder aloud, “Do you think we could create an expression for every number from 1 to 25?”

Explore:

In their groups, students can explore for 15 or more minutes. Gather the class to review the activity instructions. “In this activity, you will use the digits 1, 2, 3, and 4, in any order to create arithmetic expressions with different numeric values.

For this problem, a “1-2-3-4 expression” is any expression that uses each of these digits exactly once using” (Student Ed pg. 18)

- Any of the four basic arithmetic operations: addition, subtraction, multiplication, and division
- Exponents
- Juxtapose two digits aka put two digits next to each other
- Use square roots
- Use factorials
- Use parenthesis

Reading the instructions will give students more ideas about operations they can use. Many students won't have thought to use a square root or be familiar with factorials.

Share:

Students will be interested in the discussion of this activity in order to see expressions for numbers they haven't yet figured out. Have volunteers share expressions for answers that other students haven't found.

Summarize:

During the discussion, the teacher will clarify order-of-operations rules. As situations present themselves, have the class rewrite the solution in conventional form.

Questions you might ask about how they found their expressions include:

- What methods did you use to find your expressions?
- Did you proceed in numeric order or did you jump around?
- Did you get an expression for one number by adjusting the expression for another?
- Did you use any patterns that you saw in the expressions?

Tell students that arithmetic problems are worked out according to these rules:

- Simplify expressions within parentheses before combining them with expressions outside the parentheses.
- Within parentheses (or where no parentheses exist), do operations in this order:
 - Apply exponents to their bases.
 - Multiply and divide as the operations appear from left to right. (Neither operation has precedence over the other.)

- Add and subtract as the operations appear from left to right. (Neither operation has precedence over the other.)

Lesson 4 - Uncertain Answers (pg. 19)

Objective: Students will gain insight into the need for rules for order of operations and establish the conventional rules for order of operations.

Launch: The teacher should review the rules for order of operations: Parentheses, Exponents, Multiplication and division (equal priority) from left to right, addition and subtraction (equal priority) from left to right.

Explore:

A. Fix these equations: None of these statements are correctly written. Rewrite each, inserting parentheses in the expressions on the left so that the resulting statements are correct equations.

a. $12-8*2+7=36$

b. $8-15+6 \div 3=1$

c. $7+3^2=100$

d. $24+16 \div 8-4=10$

e. $20 \div 7-2+5^2*3=79$

B. What could it be?: Place parentheses in different places in these expressions to see how many different values you can make for each expression. Find at least 3 values for each problem.

a. $7-5*8+6 \div 2$

b. $4+9-6 \div 2*5+1$

c. $4-3-2+1$

Share:

Give groups a few minutes to share their work on the assignment. Students should be able to resolve each other's difficulties within this group discussion.

Summarize:

If you see common errors as you circulate among groups, the teacher should draw the class together for clarification. Clear up any conflicts by having students go through the problem one small step at a time. The teacher should review the rules for order of operations: Parentheses, Exponents, Multiplication and division (equal priority) from left to right, addition and subtraction (equal priority) from left to right.

Lesson 5 - Chefs' Hot and Cold Cubes (pg. 21-23)

Objective: Students will discuss the need to justify solutions when doing integer arithmetic and review notation, language, and conventions. Students will be introduced a “hot and cold cubes” model and perform integer arithmetic to help make sense of the model.

Launch: The teacher poses two questions to determine students' prior knowledge:

- What is the answer to $(-3)(-5)$? How do you know your answer is right?
- What is the answer to $-3 + -5$? How do you know your answer is right?

Record all solutions on the board. Remind students that they should be able to state why their answers are correct. Some students may be able to apply the rules to find the correct answers, but many will have trouble explaining why the product of two negative numbers is positive while the sum of two negative numbers is negative.

“Review the notation and terminology of positive and negative numbers. These activities use the “raised sign” notation, such as $+5$ and -7 . These should be read as “positive five” and “negative seven” not “plus five” or “minus seven.” Using clearly defined terminology helps to distinguish between positive and negative *numbers* and the *operations* of addition and subtraction.” (Teacher Guide “The Chefs' Hot and Cold Cubes”)

Also review the following terminology and notation with students.

- The *sign* of a number indicates whether it is positive or negative. Zero is considered neither positive nor negative.
- The numbers in a pair such as $+3$ and -3 are sometimes called *opposites*. That is, -3 is the opposite of $+3$, and $+3$ is the opposite of -3 .
- The word integer refers to a number that is zero, positive whole numbers, or negative whole numbers. $\{ \dots, -3, -2, -1, 0, +1, +2, +3, \dots \}$
- The number line is a way to picture both positive and negative numbers. Positive numbers are on the right and negative numbers are on the left; numbers are considered to get larger as one moves to the right on the number line. Thus, for example, $+5 > -8$ and $-7 < -3$.

Have students read the introduction to *The Chefs' Hot and Cold Cubes* (Student Ed. pg. 21) and the first five paragraphs of The Story.

Introduce students to the manipulatives—two colors of cubes—for representing hot and cold cubes. Ask groups to use their manipulatives to create several cauldrons, each representing a temperature of 0° , to introduce the idea that a hot cube and a cold cube “cancel out” one another.

Explore:

Have groups read the next paragraph (beginning “For each hot cube . . .”) and then create cauldrons for other specific temperatures, such as $+5^\circ$ or -3° . Students should get a sense of the cancellation mechanism and see that a given temperature can be represented in many ways.

Let students read the rest of “The Story” individually and then work in their groups on the questions.

Share:

As groups finish, have them share their answers to the questions. As they share their work, you may have to emphasize that the equations and arithmetic expressions focus on the *change* in temperature and not on the temperature itself.

Summarize:

Review that there results that the equations show the change in temperature. Acknowledge the confusion caused by the same notation being used in different ways. For instance, $+5$ can mean “add five bunches of a certain number of hot or cold cubes” (as in $+5 \cdot 20 = +100$) or “a bunch containing five hot cubes.” It is similar to the dual meaning in multiplication of whole numbers, in which $5 \cdot 3$ can mean “5 groups with 3 objects in each group” or “3 groups with 5 objects in each group,” with 5 representing either the number of groups or the size of each group.

Close by addressing the initial probing questions:

- What is the answer to $(-3)(-5)$? How do you know your answer is right?
- How might you represent the situation with objects?

Lesson 6 - Checkerboard Squares (p. 25)

Objective:

Students will produce a method of counting a closed set of information and generalize this pattern to produce a recursive formula.

Launch:

Students are asked to generalize their methods for counting the number of squares of different sizes on an 8-by-8 checkerboard to produce a method for counting the squares on a board with dimensions n by n . You could begin by having an 8-by-8 checkerboard and showing the students a few examples of what would work and have them offer a few of their own solutions.

Explore:

Allow students to work in pairs or groups to begin counting all of the possible number of squares in an 8-by-8. Have them devise a way to keep all of their findings organized. If students are starting to find the pattern, encourage them to try a 9-by-9 and eventually an n -by- n .

Share:

Allow students to view the work of other groups in their class. As they read other students' work, you might have students focus on what makes a good write-up, what makes an adequate write-up, and what makes a poor write-up. Have assigned students give presentations, limiting each to about five minutes. Encourage presenters to speak about their investigation process at least as much as they speak about their findings.

In the discussion that grows out of the presentations, focus on the patterns that students have discovered. Bring out that finding patterns helps us to analyze mathematical situations.

Summarize:

Student interest may offer opportunities to extend the exploration. For example, this activity lends itself to trying to explain *why* the square numbers appear. Students, or you, may raise such questions as these: *Why is the number of squares of each size itself a square number? Why is it the particular square that it is?*

Lesson 7 - Consecutive Sums (p. 28)

Objective:

Students will explore patterns of consecutive sums and represent those patterns with a conjecture.

Launch:

Introduce the idea of a consecutive number with several examples. Emphasize that consecutive means that one number is one more than the previous number. Give an example of non-consecutive numbers, such as 6, 8 and 10 and explain why they are not consecutive. Define what it means to be a consecutive sum with several examples. For this activity, use only the natural numbers.

Explore:

Have all students create one example of their own to share with their group. Once all groups have several examples to look at, have them determine if they can find any patterns in their sums and try to make some generalizations. The following are some of the questions that groups might investigate.

- What numbers can be written as consecutive sums?
- What numbers can be written as more than one consecutive sum?
- Are there patterns to the answers to consecutive sums that are two terms long (such as $4 + 5$), three terms long, or four terms long?

As groups are finishing up their exploration, they should display their findings on a poster to share with the class.

Share:

Once groups have displayed their posters, review and discuss this collection of conjectures and summary statements. Ask a member of each group to state one of the patterns that the group found that hasn't yet been mentioned. Continue until no group has summary statements that haven't already been mentioned.

It may work best to have all the statements read before getting into discussion of or challenges to any of them. When ready, invite students to comment on the summary statements of other groups. They may have facts that contradict a given statement, or they may simply question whether a given generalization is valid.

Introduce the word counterexample in the context of these summary statements by asking whether there are any cases in which a generalization doesn't hold. (If no one offers one, suggest one yourself.) For example, the summary statement "If a number can be written in three or more ways as a consecutive sum, then it must be odd" is false, and 30 is a counterexample. Although 30 fits the condition that "it can be written in three or more ways as a consecutive sum," it doesn't have the property "it must be odd."

On the basis of this discussion, the class may eliminate or confirm some of the summary statements, while others will remain conjectures.

Summarize:

Following are some possible summary statements.

- Every odd number greater than 1 can be written as a consecutive sum of two terms. (This particular statement is the subject of the activity *That's Odd!*) Because only positive whole numbers are permitted in the activity, 1 itself cannot be written as a consecutive sum.
- The numbers 1, 2, 4, 8, 16, . . . (powers of 2) cannot be written as consecutive sums.
- The numbers that cannot be written as consecutive sums are all even. (This statement is incorrect, because 1 is odd but cannot be written as a consecutive sum. It can be written as $0 + 1$, but the activity allows only positive terms, not 0.)
- Every third number—that is, every multiple of 3—except 3 itself can be written as a consecutive sum of three terms. (The number 3 is $0 + 1 + 2$, but again, 0 is not permitted.)

Lesson 8 - Add It Up (p. 30)

Objective:

Students will use summation notation with both numeric and geometric examples.

Launch:

Introduce the activity with a multiterm example of a [consecutive sum](#), such as $3 + 4 + 5 + 6 + 7 + 8 + 9$. Demonstrate that there is a shorthand way for writing such sums: [example] . Explain that this symbol is an uppercase letter in the Greek alphabet, called *sigma*, and that the expression is read, “The summation, from i equals 3 to 9, of i .” Invite students to articulate the connection between the shorthand and the full expression. Use a more complex example to illustrate in detail how this notation works. It might even be helpful to have students “act” out the process.

Explore:

Allow students to work in groups on 1a,b and c and try to come up with a sum. Once they are done have them put their answers on the board. Discuss as a class how each group came up with their answers.

Allow groups to continue on the rest of the problems using summation notation.

Share:

Allow students time to share their responses and ask questions of other groups.

Summarize:

Once students have had several chances to work with summations and see how others use them, discuss how summations can be useful in math.

Lesson 9 - That's Odd (p. 33)

Objective:

Students will express odd numbers algebraically and begin the process of writing proofs.

Launch:

Introduce the problem statement: *If an odd number is greater than 1, then it can be written as the sum of two consecutive numbers.*

Ask students if they think the statement is true or false and how do they know? Give them some examples, such as 397 and 4913, and ask how they could write them as consecutive numbers. Read the instructions and clarify any confusion.

Explore:

Students should either be finding a “counterexample” or creating a general set of instructions showing why it works.

Share:

Begin the discussion by asking the class again whether they think the conjecture is true. Then ask how confident they are that it is true for *every* odd number greater than 1. Most students will likely be fairly sure that it is always true, but encourage skeptics to voice their opinions.

Ask for volunteers to share any instructions they developed for writing an odd number as a sum of two consecutive numbers, and have them illustrate their methods using specific examples. If the class is at a loss about how to do this, you might ask a series of questions, such as, **How would you write 397 as the sum of two consecutive numbers? How would you write 4913 as the sum of two consecutive numbers? How would you write 157,681 as the sum of two consecutive numbers?** Encourage students to explain how to find the pair of consecutive integers in each case, as this is key to developing a general argument.

There are several ways to describe the general process; elicit as many as possible from your students. Here are some commonly suggested procedures.

- Subtract 1 from the odd number to get an even number. Divide this even number by 2. That quotient and the next number are the desired consecutive numbers.
- Divide the odd number by 2, getting “something and a half.” The whole numbers just above and below this mixed number are the desired consecutive numbers.
- Add 1 to the odd number to get an even number. Divide this even number by 2. That quotient and the previous number are the desired consecutive numbers.

Careful examination of any of these methods will show that they don't work if the initial odd number is 1, because one of the numbers in the [consecutive sum](#) will be 0 rather than a positive [whole number](#) as required.

Whichever methods students suggest, ask them to explain how they know that a given method works. For example, for the first procedure above, you might ask how students know that subtracting 1 from an odd number gives an even number. The best response to this question would refer to a definition of the [term](#) *odd*. That is, students should recognize that, ultimately, they can't say anything for sure about odd numbers unless they begin with a clear definition. Similarly, ask how students know that dividing an even number by 2 gives a whole-number result. Again, encourage them to see that the answer to this challenge depends on having a precise definition of the term *even*. It is not necessary to go into formalities about the meaning of the terms *odd* and *even*. What is important is recognizing the value of having a precise definition if one is to give a complete proof.

Summarize:

Use the discussion to help bring out the difference between a collection of examples of a phenomenon and a legitimate general proof. A proof does not need to use algebraic symbols. For example, when appropriate and precise definitions are given for *odd* and *even*, the arguments above constitute completely legitimate proofs that every odd number can be written as a consecutive sum with two terms. Help students to see that these arguments are better than only giving a few examples such as $23 = 11 + 12$ and $47 = 23 + 24$.

Each procedure listed above demonstrates that every odd number is expressible as a consecutive sum of two terms by showing *how to do it*, that is, how to find the two terms. Such how-to arguments are considered legitimate proofs and are known as *constructive proofs*.

Algebraic symbols do sometimes help students understand a situation, and your students may be able to express their arguments symbolically. For example, if you suggest using n for the number obtained after subtracting 1 and dividing by 2, students can probably write the next number as $n + 1$.

Lesson 10 - A Fractional Life

Objective:

Students will perform several operations on fractions to solve a real-world problem.

Launch:

Teacher could expand upon Greek Anthology (not necessary, but may introduce some ideas of how mathematics once was). Ask students to think about someone that has lived a long life and what stages they went through to get there. Some examples may include: How long were they a baby? How about a child?...

Another thought may include the idea of how long is one life? Ask groups to reflect upon this until one comes up with it.

Explore:

Allow groups to work on the mathematical portion of this activity. As you circulate, you may need to remind groups of how to add fractions together by getting common denominators.

Share:

Once groups start to get the answer, have them write their results on the board. Check to see if all groups come to same answer and ask certain groups to explain their reasoning.

Summarize:

The big take-away in this problem would be to identify that all the pieces of the puzzle are all part of the same variable (the age of Demochares) and that common denominators are needed to complete it.

Lesson 11 - Chef Divisions

Objective:

Students will use the principles from “The Chef’s Hot and Cold Cubes” to determine a rule for division.

Launch:

Remind students of the Chef’s Hot and Cold Cubes and what operations the Chef used. Also have students remember all of the “Order of Operations” and discuss what operations were left out. Then reflect on the fact that division was left out and what it could mean if the Chef could divide.

Explore:

Students should work in groups to define problem 1 and work on a problem of their own.

Share:

When groups are ready, have them present what they think the division sign means in terms of the Chef and an example of their own. Have other groups reflect on each group’s ideas and determine and correct idea.

Summarize:

As groups determine which should be acceptable, it is important for the teacher to emphasize that a/b needs to result in a whole number and that is why division was left out due to “invalid” solutions.

Lesson 12 - From 1 to N

Objective:

Students will express the terms in a sequence with a closed formula.

Launch:

Ask students to recall that add what sum of adding the numbers from 1 to 4 was. Then ask them to do it again with the number 1 to 9. Tell them that their task is to find a simple expression in terms of n that allows us to find the sum of the numbers:

$$1 + 2 + \dots + n.$$

Explore:

Students will work individually to find a way to represent this sum with a formula. After a while allow students to share what they have with their groups. Allow groups to try to solidify one possible answer. Once groups have their expressions, ask them to look for a proof to ensure their answer is correct, meaning it works for every value of n .

Share:

Have one or two groups share their proofs and show how they can find the sum for any value of n .

Summarize:

Double check that groups are writing their proofs effectively and negate any confusions. At this point, students should have a good understanding of consecutive numbers and summations.

Short Answer.

1. Explain each of the following problems in terms of the model of hot and cold cubes. Each explanation should include a statement of how the temperature changes overall.

a) $-5 - +8$

b) $-3 \cdot + 2$

2. Does $-4 - +7 = -4 + -7$? Explain why or why not using the hot and cold cubes.

3. Write each expression as a sum of terms.

a) $\sum_{i=1}^4 -2i + 3$

b) $\sum_{r=8}^{12} (r - 7)^2$

4. Find the missing entries for the in-out table. Describe the function represented by the table both in words and by using algebraic expression.

In	Out
4	7
8	15
-2	-5
10	
-5	
	31

5. Examine the first several terms in each sequence. Look for a pattern that explains how the sequence is formed. Then write a description of the pattern and a method for finding the next three terms and write what they are.

a) 1, 4, 9, 16, ...

b) 2, 6, 18, 54, ...

c) 1, 4, 7, 10, ...

6.

Squares are stacked in piles of different heights.

Height of the stack	Number of squares
1	1
2	3
3	6
4	10

- Use diagrams or a continuation of the table to find the number of squares in a 7-high stack.
- How many squares are in a 40-high stack?
- Give a general description for how to find the number of squares in an n -high stack.

7. Simplify the following expressions:

a)

$$7 - (5 \cdot 8) + 6 \div 2$$

b)

$$20 \div 7 - 2 + 5^2 \cdot 3$$

c)

$$(4 + 5) - 6 \cdot (-3) + 4^3$$

8. Show that the square of every odd number is odd.

Multiple Choice.

9. What summation notation would represent the following sum:

a) $\sum_{i=1}^8 n^2$ b) $1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2 + 8^2$ c) $\sum_{i=1}^8 n + 1$ d) $\sum_{i=1}^8 \frac{n(n-1)}{2}$

b) $\sum_{i=0}^8 n$

10. According to the Chefs' Hot and Cold cubes what would represent taking out 5 bunches of 3 hot cubes.

a) $-5 - +3$ b) $5 - 3$ c) $-5 \cdot +3$ d) $-5 \div +3$

11. Which of the following operations, when combining a negative and a negative, result in a positive answer?

I. Addition

II. Subtraction

III. Multiplication

IV. Division

a) I only

b) II only

c) I and II

d) III and IV

12. When simplifying an expression, what operation should you perform first?

a) Addition	b) Exponents	c) Parentheses	d) Division
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